ME 141

Engineering Mechanics

Lecture 12: Kinetics of particles: Momentum method

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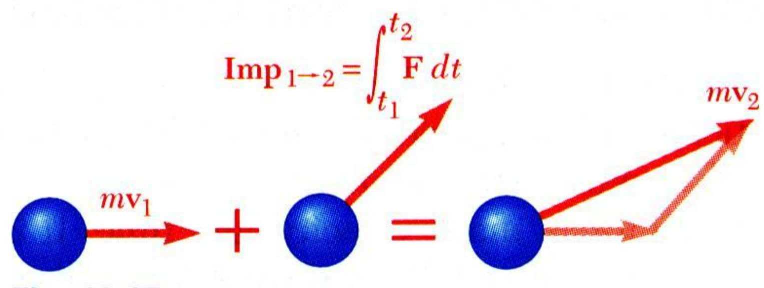
Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Impulsive Motion

The thrust of a rocket acts over a specific time period to give the rocket linear momentum.

The impulse applied to the railcar by the wall brings its momentum to zero. Crash tests are often performed to help improve safety in different vehicles.

Principle of Impulse and Momentum



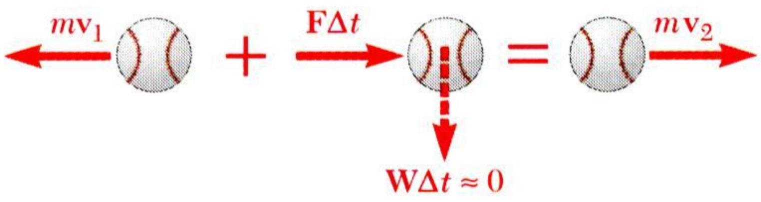
- Dimensions of the impulse of a force are
  *force*time.*

- Units for the impulse of a force are
  $$\mathrm{N}\cdot\mathrm{s} = \left(\mathrm{kg}\cdot\mathrm{m}/\mathrm{s}^2\right)\cdot\mathrm{s} = \mathrm{kg}\cdot\mathrm{m}/\mathrm{s}$$

- From Newton's second law,
  $$\vec{F} = \frac{d}{dt}(m\vec{v}) \qquad m\vec{v} = \text{linear momentum}$$
  $$\vec{F}dt = d(m\vec{v})$$
  $$\int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2 - m\vec{v}_1$$
  $$\int_{t_1}^{t_2} \vec{F}dt = \mathbf{Imp}_{1\to 2} = \text{impulse of the force } \vec{F}$$
  $$m\vec{v}_1 + \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

- The final momentum of the particle can be obtained by adding vectorially its initial momentum and the impulse of the force during the time interval.

Impulsive Motion



- Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum is called an *impulsive force*.

- When impulsive forces act on a particle,

$$m\vec{v}_1 + \sum \vec{F}\,\Delta t = m\vec{v}_2$$

- When a baseball is struck by a bat, contact occurs over a short time interval but force is large enough to change sense of ball motion.

- *Nonimpulsive forces* are forces for which $\vec{F}\Delta t$ is small and therefore, may be neglected – an example of this is the weight of the baseball.

Sample Problem 13.10

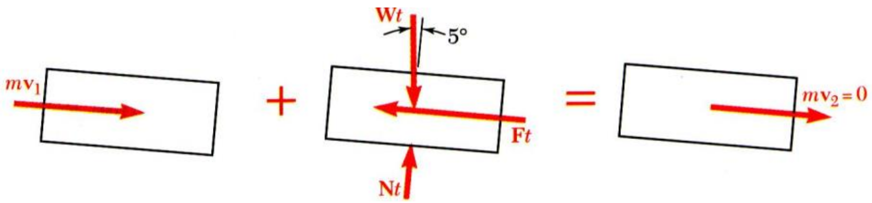
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied, causing a constant total braking force of 1500 lb.

Determine the time required for the automobile to come to a stop.

SOLUTION:

- Apply the principle of impulse and momentum. The impulse is equal to the product of the constant forces and the time interval.

Sample Problem 13.10



SOLUTION:

- Apply the principle of impulse and momentum.

$$m\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

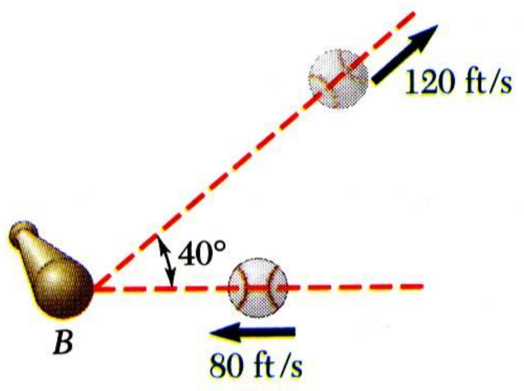
Taking components parallel to the incline,

$$mv_1 + (W \sin 5°)t - Ft = 0$$

$$\left(\frac{4000}{32.2}\right)(88\,\mathrm{ft/s}) + (4000 \sin 5°)t - 1500t = 0$$

$$\boxed{t = 9.49\,\mathrm{s}}$$

Sample Problem 13.11

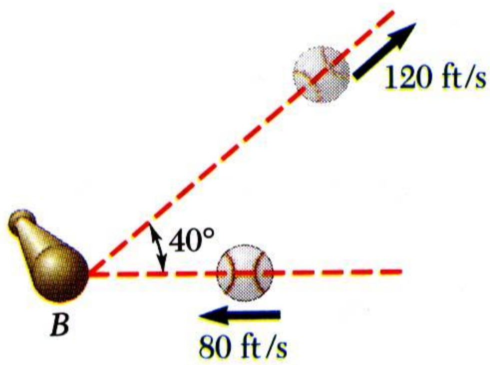


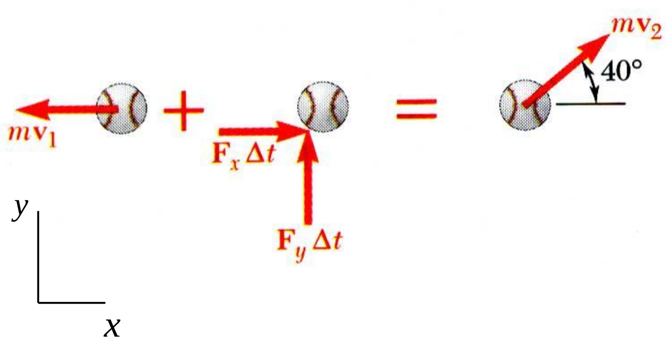
A 4 oz baseball is pitched with a velocity of 80 ft/s. After the ball is hit by the bat, it has a velocity of 120 ft/s in the direction shown. If the bat and ball are in contact for 0.015 s, determine the average impulsive force exerted on the ball during the impact.

SOLUTION:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

Sample Problem 13.11





SOLUTION:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

$$m\vec{v}_1 + \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

x component equation:

$$-mv_1 + F_x\Delta t = mv_2 \cos 40^\circ$$

$$-\frac{4/16}{32.2}(80) + F_x(0.15) = \frac{4/16}{32.2}(120\cos 40^\circ)$$

$$F_x = 89\,\text{lb}$$

y component equation:

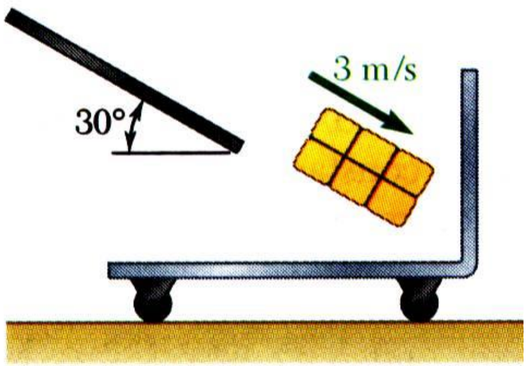
$$0 + F_y\Delta t = mv_2 \sin 40^\circ$$

$$F_y(0.15) = \frac{4/16}{32.2}(120\cos 40^\circ)$$

$$F_y = 39.9\,\text{lb}$$

$$\boxed{\vec{F} = (89\,\text{lb})\vec{i} + (39.9\,\text{lb})\vec{j}, \quad F = 97.5\,\text{lb}}$$

Sample Problem 13.12



A 10 kg package drops from a chute into a 24 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.

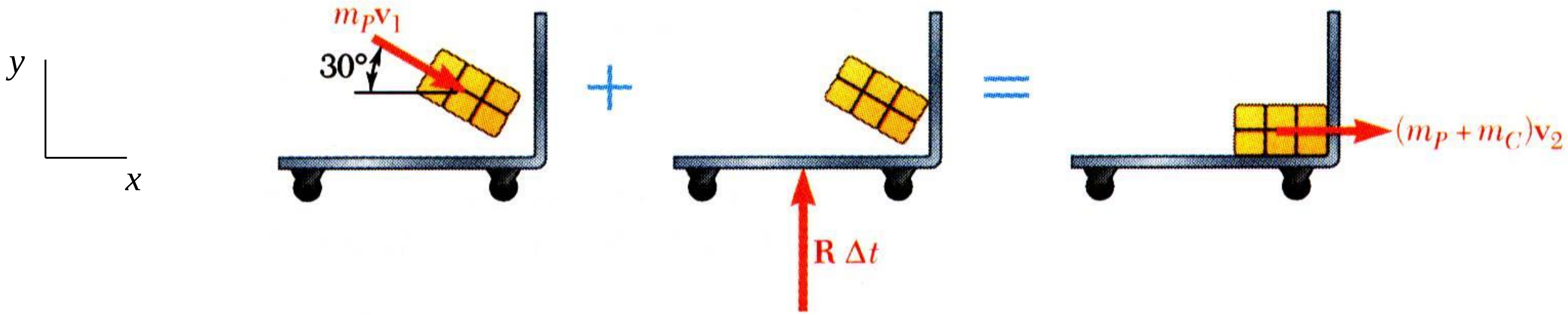
<u>SOLUTION</u>:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.
- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.

Sample Problem 13.12

<u>SOLUTION</u>:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.



$$m_p \vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = (m_p + m_c)\vec{v}_2$$

x components:

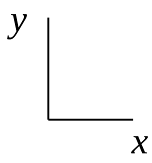
$$m_p v_1 \cos 30° + 0 = (m_p + m_c)v_2$$

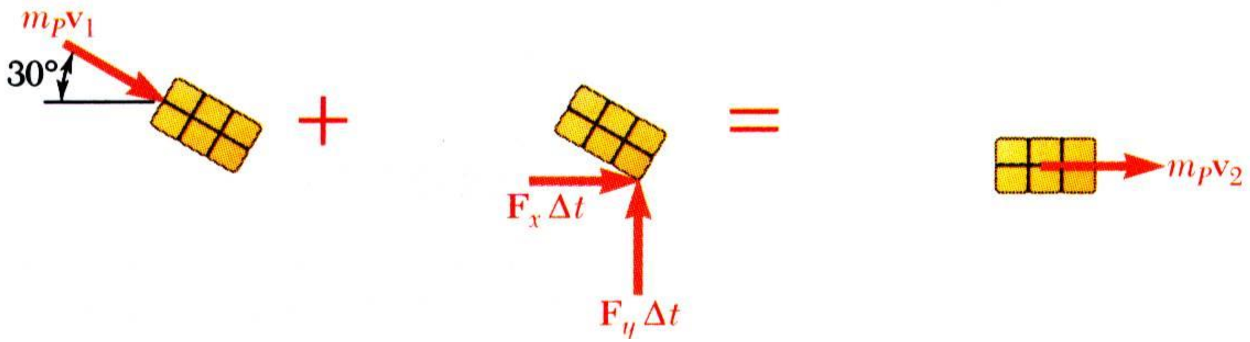
$$(10\text{ kg})(3\text{ m/s})\cos 30° = (10\text{ kg} + 25\text{ kg})v_2$$

$$\boxed{v_2 = 0.742\text{ m/s}}$$

Sample Problem 13.12

- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.





$$m_p\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m_p\vec{v}_2$$

x components:

$$m_p v_1 \cos 30° + F_x\Delta t = m_p v_2$$

$$(10\text{ kg})(3\text{ m/s})\cos 30° + F_x\Delta t = (10\text{ kg})v_2 \qquad F_x\Delta t = -18.56\text{ N}\cdot\text{s}$$

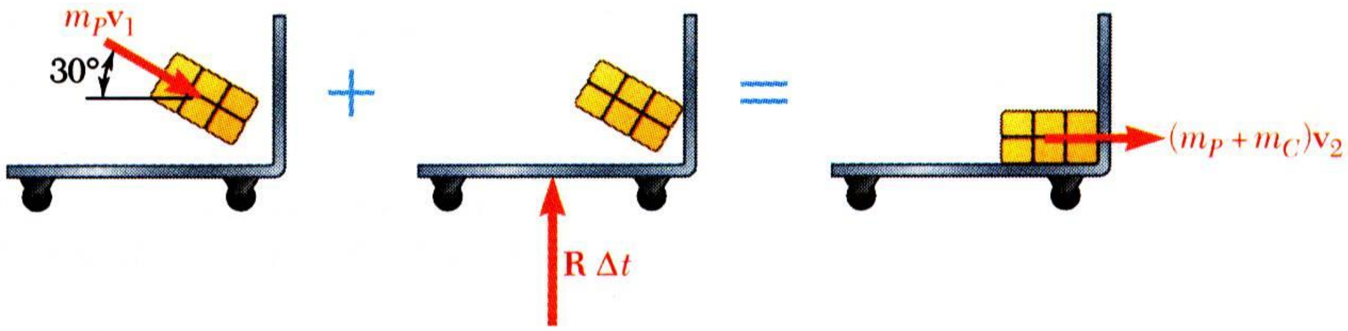
y components:

$$-m_p v_1 \sin 30° + F_y\Delta t = 0$$

$$-(10\text{ kg})(3\text{ m/s})\sin 30° + F_y\Delta t = 0 \qquad F_y\Delta t = 15\text{ N}\cdot\text{s}$$

$$\boxed{\sum \mathbf{Imp}_{1\to 2} = \vec{F}\Delta t = (-18.56\text{ N}\cdot\text{s})\vec{i} + (15\text{ N}\cdot\text{s})\vec{j} \qquad F\Delta t = 23.9\text{ N}\cdot\text{s}}$$

Sample Problem 13.12



To determine the fraction of energy lost,

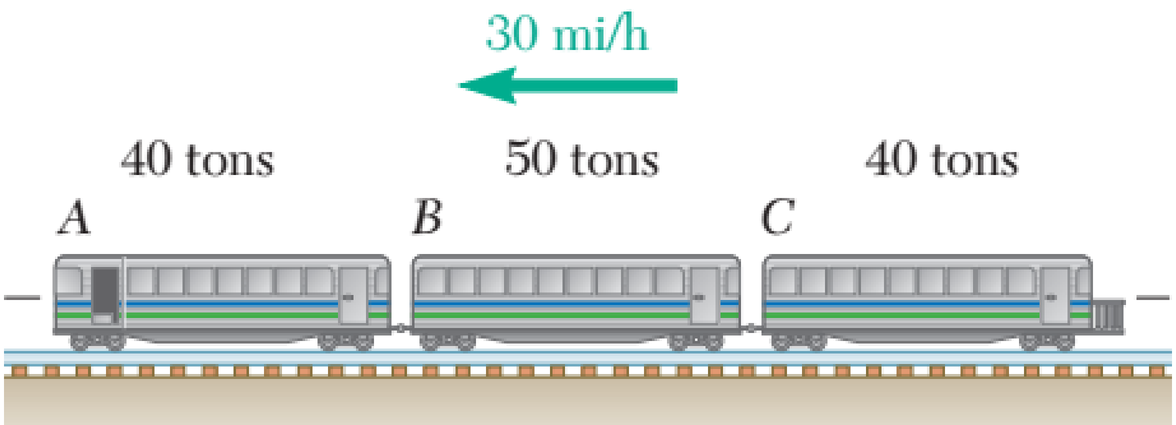
$$T_1 = \tfrac{1}{2} m_p v_1^2 = \tfrac{1}{2}(10\text{ kg})(3\,\text{m/s})^2 = 45\text{ J}$$

$$T_2 = \tfrac{1}{2}(m_p + m_c)v_2^2 = \tfrac{1}{2}(10\text{ kg} + 25\text{ kg})(0.742\,\text{m/s})^2 = 9.63\text{ J}$$

$$\boxed{\frac{T_1 - T_2}{T_1} = \frac{45\text{ J} - 9.63\text{ J}}{45\text{ J}} = 0.786}$$

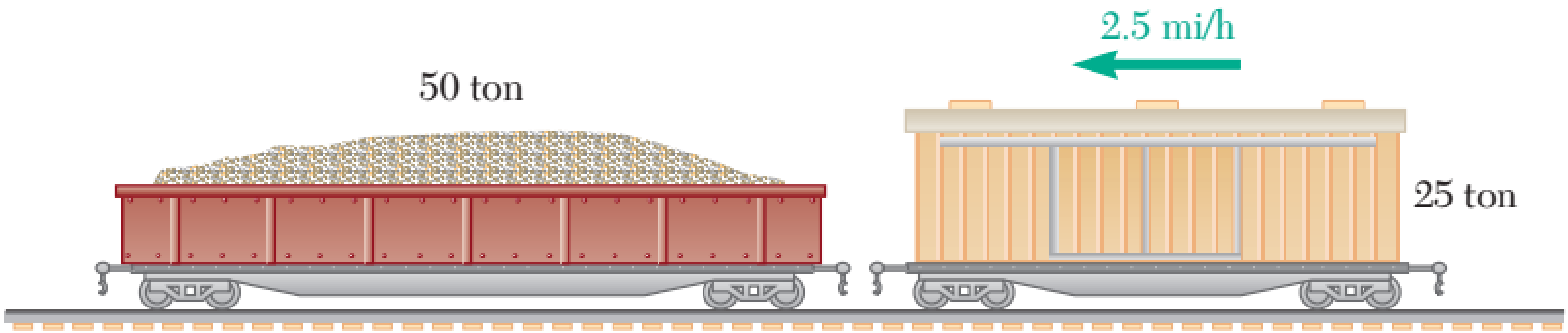
Prob# 13.129

The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *B* and *C*, causing them to slide on the track, but are not applied on the wheels of car *A*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track,
determine (*a*) the time required to bring the train to a stop, (*b*) the force in each coupling.



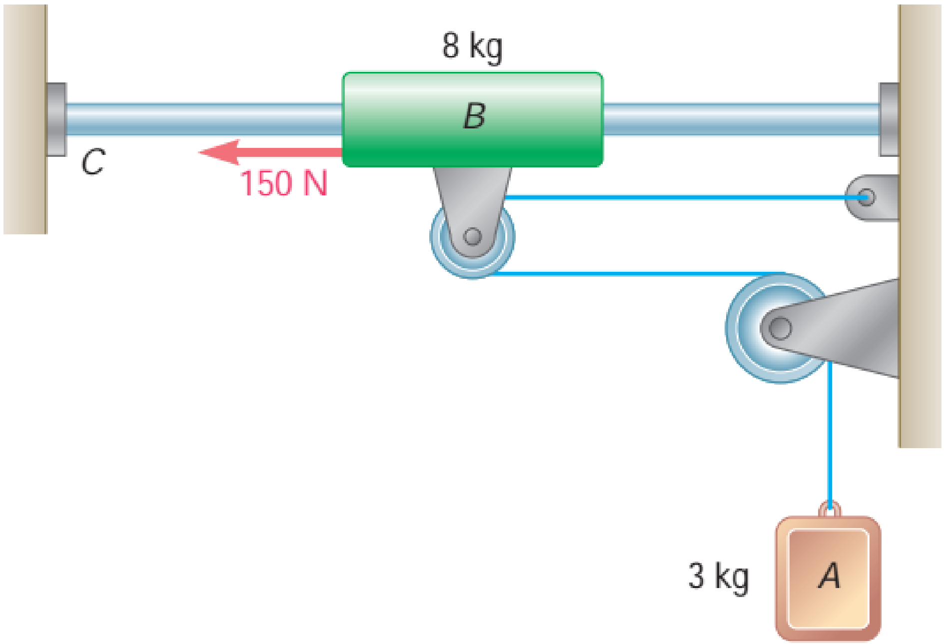
Prob# 13.145

A 25-ton railroad car moving at 2.5 mi/h is to be coupled to a 50-ton car which is at rest with locked wheels (μ_k =0.30). Determine (*a*) the velocity of both cars after the coupling is completed, (*b*) the time it takes for both cars to come to rest.

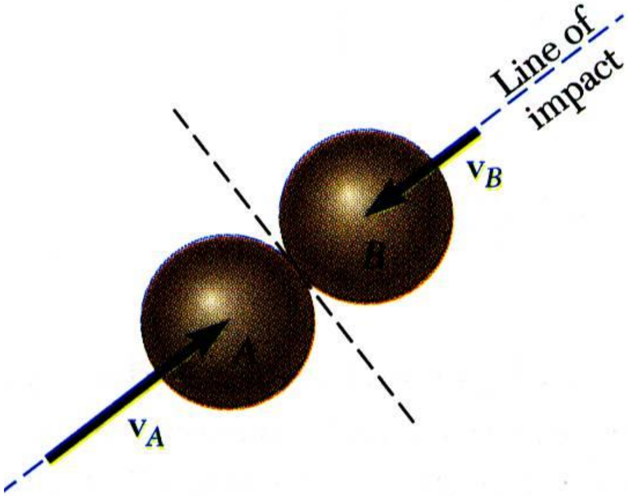


Prob# 13.132

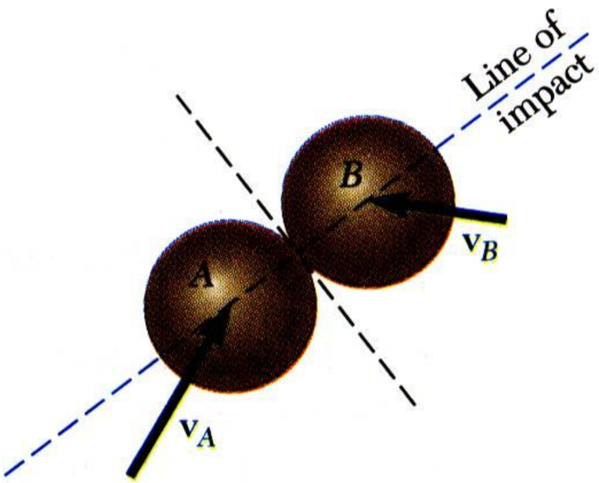
The system shown is at rest when a constant 150-N force is applied to collar *B*. Neglecting the effect of friction, determine (*a*) the time at which the velocity of collar *B* will be 2.5 m/s to the left, (*b*) the corresponding tension in the cable.



Impact



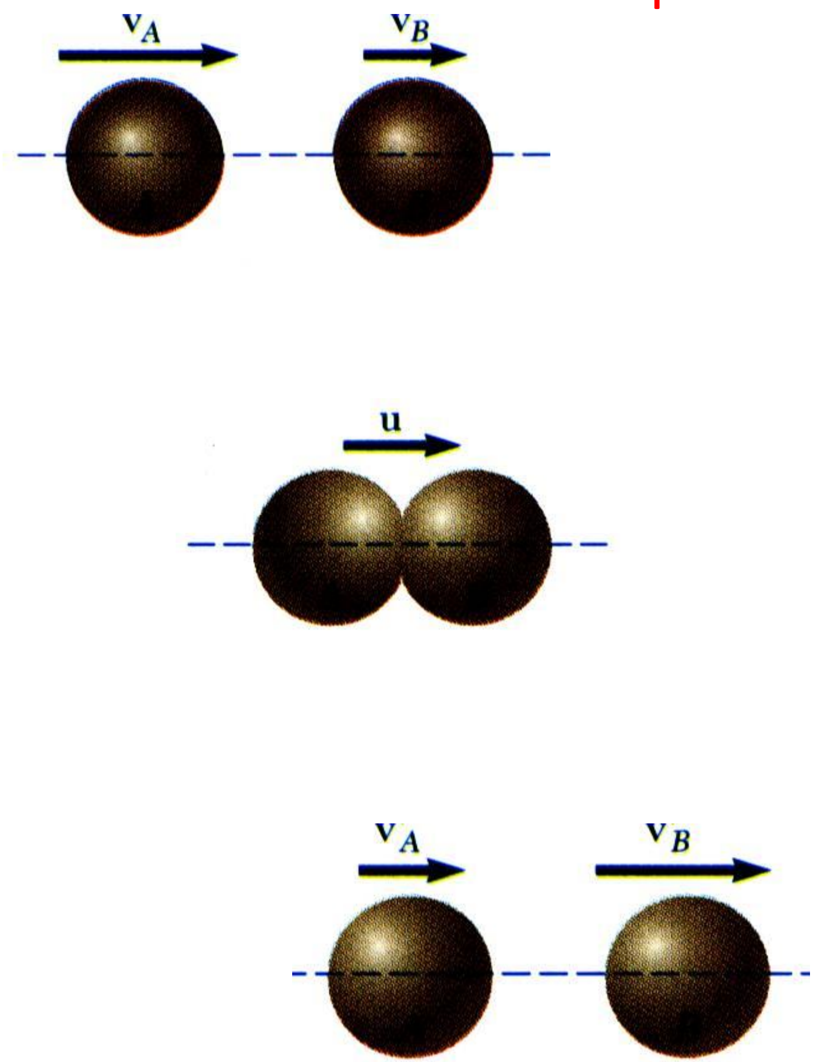
Direct Central Impact



Oblique Central Impact

- *Impact:* Collision between two bodies which occurs during a small time interval and during which the bodies exert large forces on each other.
- *Line of Impact:* Common normal to the surfaces in contact during impact.
- *Central Impact:* Impact for which the mass centers of the two bodies lie on the line of impact; otherwise, it is an *eccentric impact*..
- *Direct Impact:* Impact for which the velocities of the two bodies are directed along the line of impact.
- *Oblique Impact:* Impact for which one or both of the bodies move along a line other than the line of impact.

Direct Central Impact

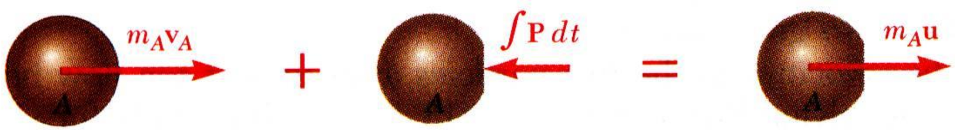


- Bodies moving in the same straight line, $v_A > v_B$.
- Upon impact the bodies undergo a *period of deformation*, at the end of which, they are in contact and moving at a common velocity.
- A *period of restitution* follows during which the bodies either regain their original shape or remain permanently deformed.
- Wish to determine the final velocities of the two bodies. The total momentum of the two body system is preserved,

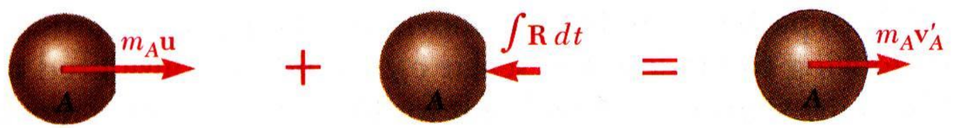
$$\boxed{m_A v_A + m_B v_B = m_A v_A' + m_B v_B'}$$

- A second relation between the final velocities is required.

Direct Central Impact



- Period of deformation: $m_A v_A - \int P dt = m_A u$



- Period of restitution: $m_A u - \int R dt = m_A v'_A$

$$\boxed{\begin{aligned} &e = \textit{coefficient of restitution} \\ &\quad = \frac{\int R dt}{\int P dt} = \frac{u - v'_A}{v_A - u} \\ &0 \le e \le 1 \end{aligned}}$$

- A similar analysis of particle *B* yields

$$e = \frac{v'_B - u}{u - v_B}$$

- Combining the relations leads to the desired second relation between the final velocities.

$$\boxed{v'_B - v'_A = e(v_A - v_B)}$$

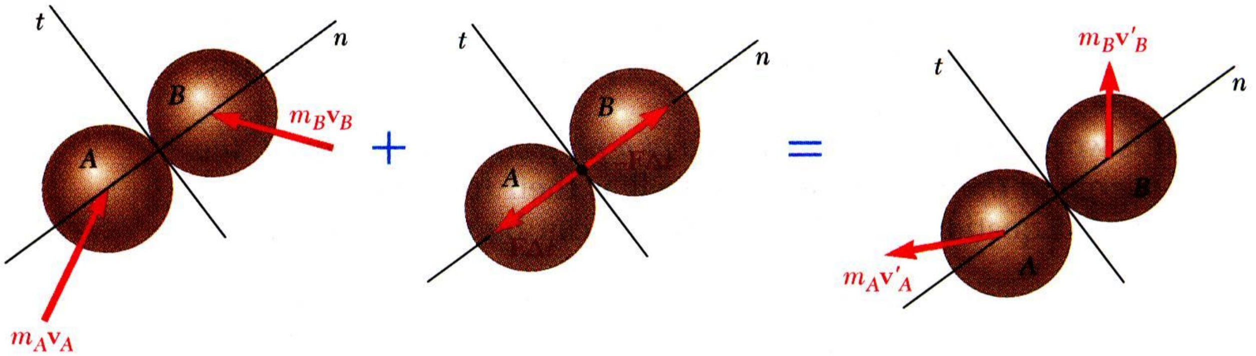
- *Perfectly plastic impact, e* = 0: $v'_B = v'_A = v'$

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

- *Perfectly elastic impact, e* = 1:
  Total energy and total momentum conserved.

$$v'_B - v'_A = v_A - v_B$$

Oblique Central Impact



- Final velocities are unknown in magnitude and direction. Four equations are required.

- No tangential impulse component; tangential component of momentum for each particle is conserved.

$$\boxed{(v_A)_t = (v'_A)_t \qquad (v_B)_t = (v'_B)_t}$$

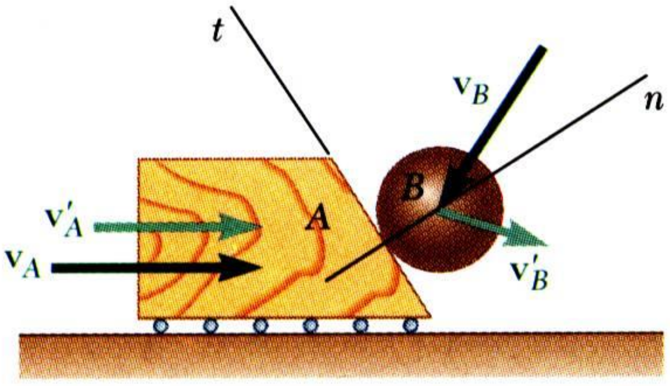
- Normal component of total momentum of the two particles is conserved.

$$\boxed{m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n}$$

- Normal components of relative velocities before and after impact are related by the coefficient of restitution.

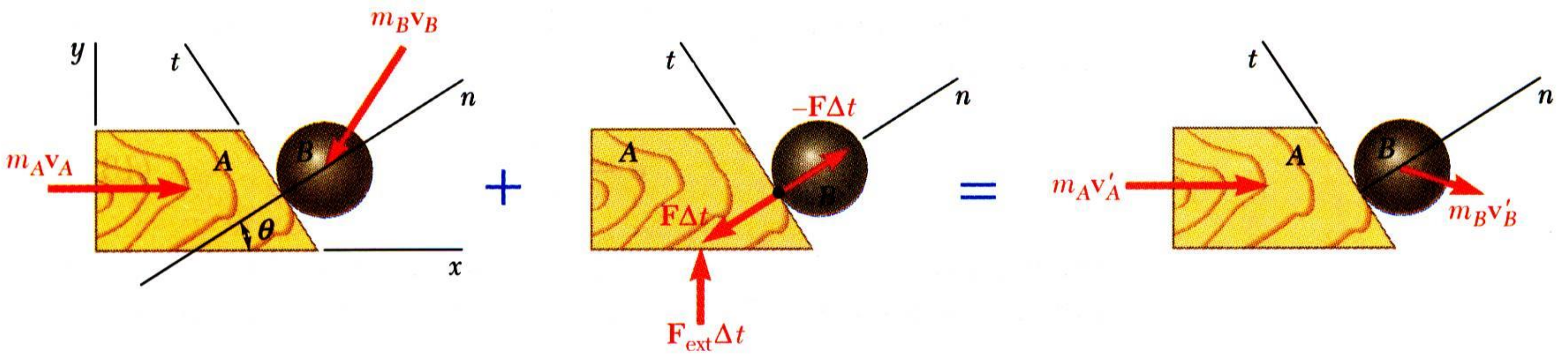
$$\boxed{(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]}$$

Oblique Central Impact



- Block constrained to move along horizontal surface.
- Impulses from internal forces $\vec{F}$ and $-\vec{F}$ along the n axis and from external force $\vec{F}_{ext}$ exerted by horizontal surface and directed along the vertical to the surface.
- Final velocity of ball unknown in direction and magnitude and unknown final block velocity magnitude. Three equations required.

Oblique Central Impact



- Tangential momentum of ball is conserved.

$$\boxed{(v_B)_t = (v'_B)_t}$$

- Total horizontal momentum of block and ball is conserved.

$$\boxed{m_A(v_A) + m_B(v_B)_x = m_A(v'_A) + m_B(v'_B)_x}$$

- Normal component of relative velocities of block and ball are related by coefficient of restitution.

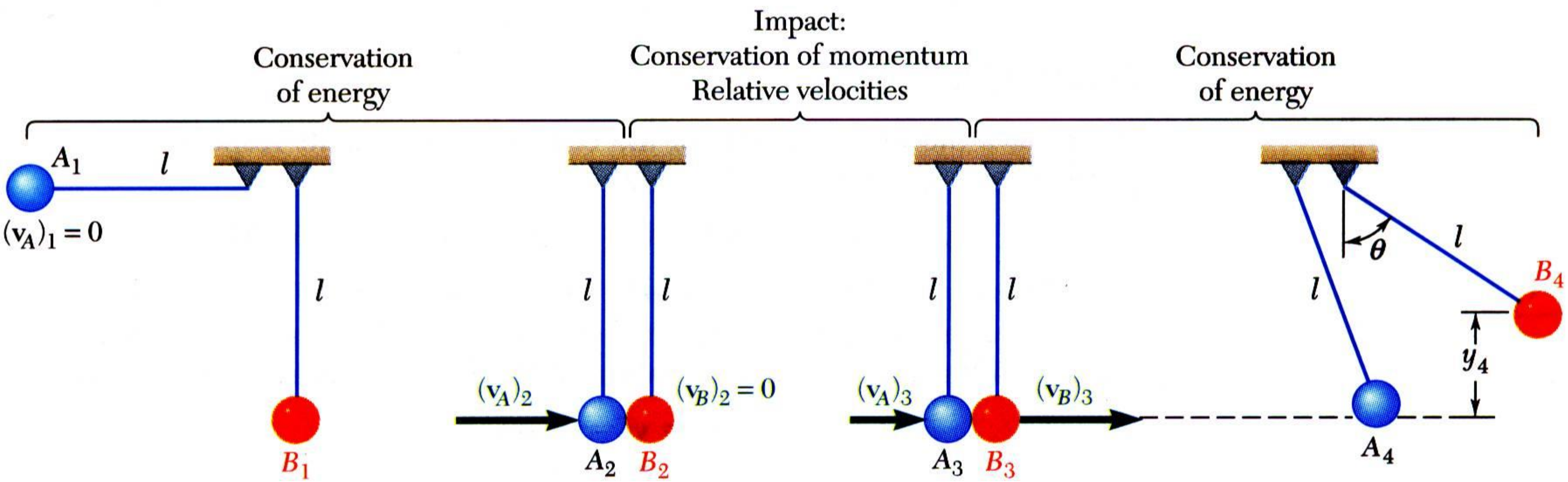
$$\boxed{(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]}$$

- Note: Validity of last expression does not follow from previous relation for the coefficient of restitution. A similar but separate derivation is required.

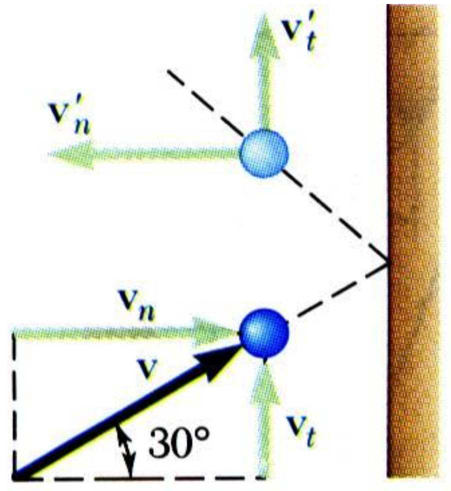
Problems Involving Energy and Momentum

- Three methods for the analysis of kinetics problems:
  - Direct application of Newton's second law
  - Method of work and energy
  - Method of impulse and momentum

- Select the method best suited for the problem or part of a problem under consideration.



Sample Problem 13.14

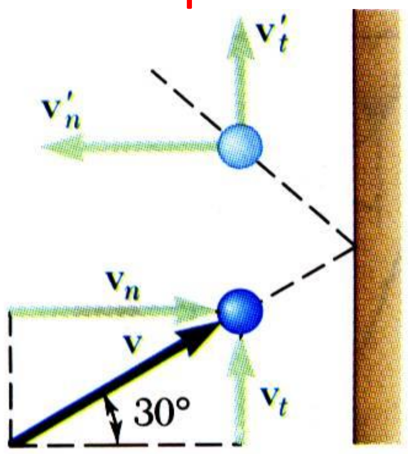


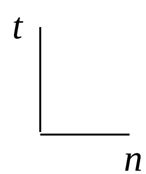
A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude v and forms angle of 30° with the horizontal. Knowing that e = 0.90, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall.

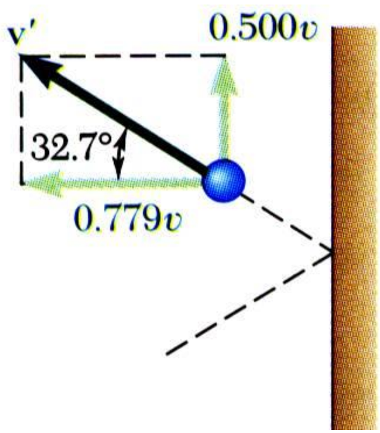
<u>SOLUTION</u>:

- Resolve ball velocity into components normal and tangential to wall.
- Impulse exerted by the wall is normal to the wall. Component of ball momentum tangential to wall is conserved.
- Assume that the wall has infinite mass so that wall velocity before and after impact is zero. Apply coefficient of restitution relation to find change in normal relative velocity between wall and ball, i.e., the normal ball velocity.

Sample Problem 13.14







<u>SOLUTION</u>:

- Resolve ball velocity into components parallel and perpendicular to wall.

$$v_n = v\cos 30° = 0.866v \qquad v_t = v\sin 30° = 0.500v$$

- Component of ball momentum tangential to wall is conserved.

$$v'_t = v_t = 0.500v$$

- Apply coefficient of restitution relation with zero wall velocity.

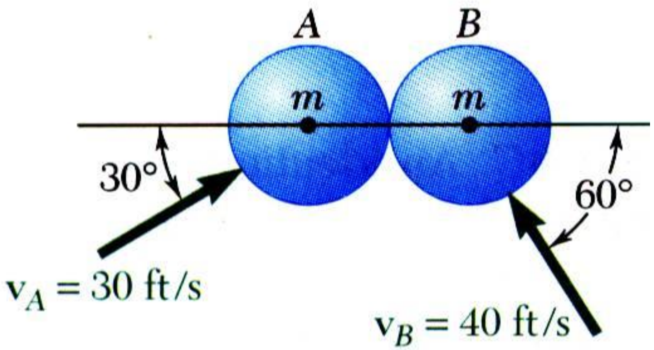
$$0 - v'_n = e(v_n - 0)$$

$$v'_n = -0.9(0.866v) = -0.779v$$

$$\vec{v}' = -0.779v\,\vec{\lambda}_n + 0.500v\,\vec{\lambda}_t$$

$$v' = 0.926v \quad \tan^{-1}\left(\frac{0.779}{0.500}\right) = 32.7°$$

Sample Problem 13.15

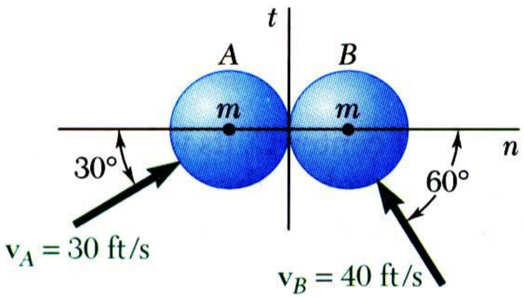


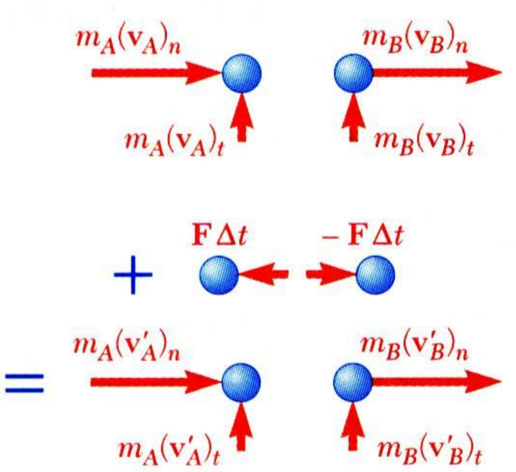
The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming $e = 0.9$, determine the magnitude and direction of the velocity of each ball after the impact.

<u>SOLUTION</u>:

- Resolve the ball velocities into components normal and tangential to the contact plane.
- Tangential component of momentum for each ball is conserved.
- Total normal component of the momentum of the two ball system is conserved.
- The normal relative velocities of the balls are related by the coefficient of restitution.
- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

Sample Problem 13.15





SOLUTION:

- Resolve the ball velocities into components normal and tangential to the contact plane.

$$(v_A)_n = v_A \cos 30° = 26.0\,\text{ft/s} \qquad (v_A)_t = v_A \sin 30° = 15.0\,\text{ft/s}$$

$$(v_B)_n = -v_B \cos 60° = -20.0\,\text{ft/s} \qquad (v_B)_t = v_B \sin 60° = 34.6\,\text{ft/s}$$

- Tangential component of momentum for each ball is conserved.

$$(v'_A)_t = (v_A)_t = 15.0\,\text{ft/s} \qquad (v'_B)_t = (v_B)_t = 34.6\,\text{ft/s}$$

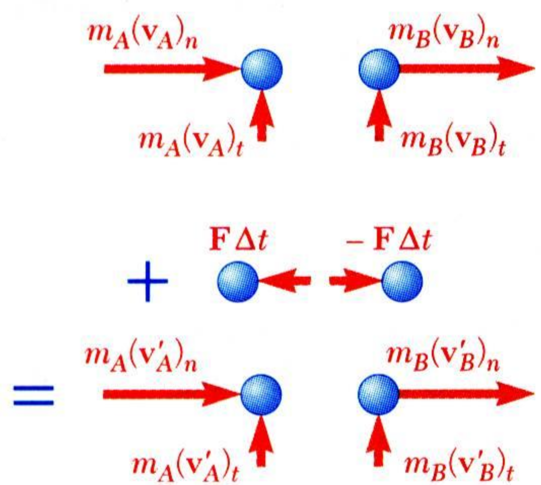
- Total normal component of the momentum of the two ball system is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$m(26.0) + m(-20.0) = m(v'_A)_n + m(v'_B)_n$$

$$(v'_A)_n + (v'_B)_n = 6.0$$

Sample Problem 13.15

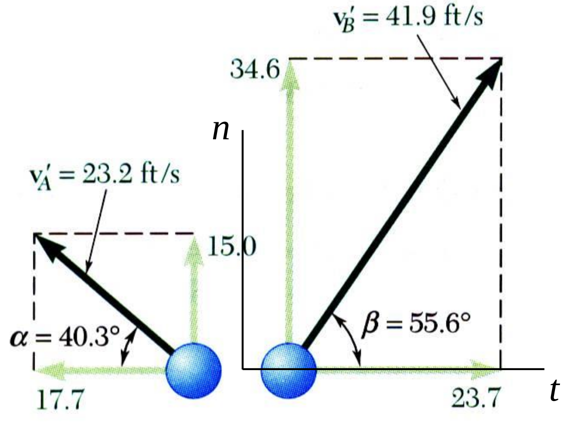


- The normal relative velocities of the balls are related by the coefficient of restitution.

$$\left(v'_B\right)_n - \left(v'_A\right)_n = e\left[\left(v_A\right)_n - \left(v_B\right)_n\right]$$
$$= 0.90\left[26.0 - (-20.0)\right] = 41.4$$

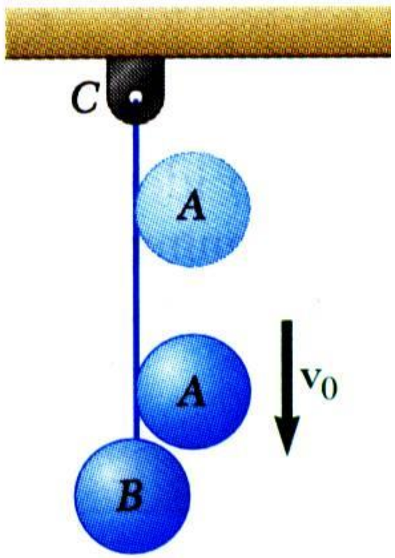
- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

$$\left(v'_A\right)_n = -17.7\,\mathrm{ft/s} \qquad \left(v'_B\right)_n = 23.7\,\mathrm{ft/s}$$



$$\vec{v}'_A = -17.7\vec{\lambda}_t + 15.0\vec{\lambda}_n$$
$$v'_A = 23.2\,\mathrm{ft/s} \qquad \tan^{-1}\left(\frac{15.0}{17.7}\right) = 40.3°$$
$$\vec{v}'_B = 23.7\vec{\lambda}_t + 34.6\vec{\lambda}_n$$
$$v'_B = 41.9\,\mathrm{ft/s} \qquad \tan^{-1}\left(\frac{34.6}{23.7}\right) = 55.6°$$

Sample Problem 13.16

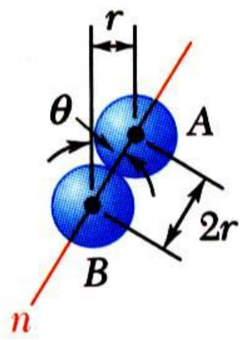


Ball *B* is hanging from an inextensible cord. An identical ball *A* is released from rest when it is just touching the cord and acquires a velocity v_0 before striking ball *B*. Assuming perfectly elastic impact ($e = 1$) and no friction, determine the velocity of each ball immediately after impact.

<u>SOLUTION</u>:

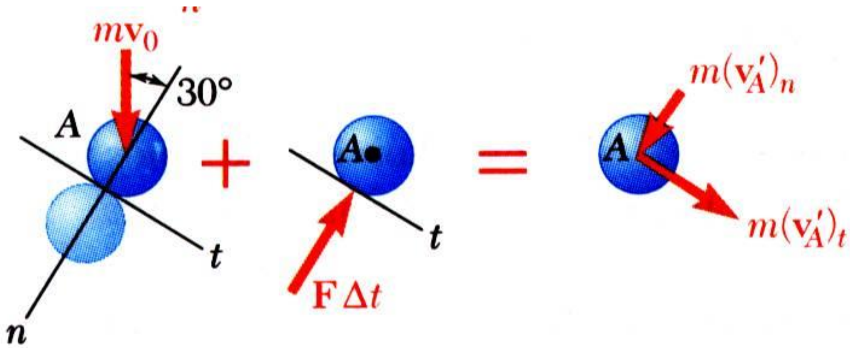
- Determine orientation of impact line of action.
- The momentum component of ball *A* tangential to the contact plane is conserved.
- The total horizontal momentum of the two ball system is conserved.
- The relative velocities along the line of action before and after the impact are related by the coefficient of restitution.
- Solve the last two expressions for the velocity of ball *A* along the line of action and the velocity of ball *B* which is horizontal.

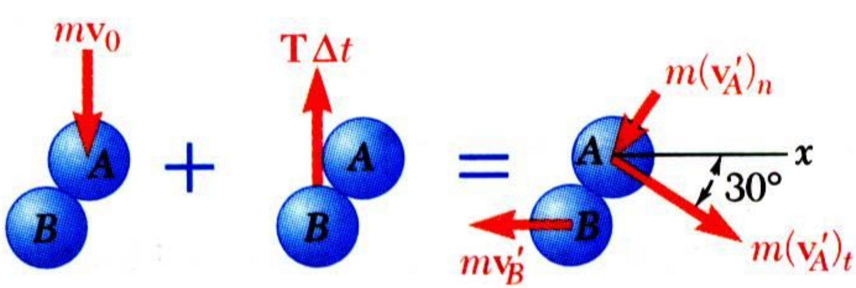
Sample Problem 13.16



$$\sin\theta = \frac{r}{2r} = 0.5$$

$$\theta = 30°$$





SOLUTION:

- Determine orientation of impact line of action.
- The momentum component of ball *A* tangential to the contact plane is conserved.

$$m\vec{v}_A + \vec{F}\Delta t = m\vec{v}'_A$$

$$mv_0 \sin 30° + 0 = m(v'_A)_t$$

$$(v'_A)_t = 0.5v_0$$

- The total horizontal (*x* component) momentum of the two ball system is conserved.

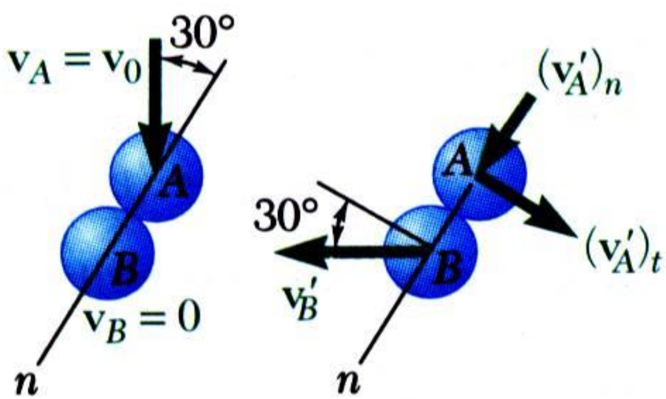
$$m\vec{v}_A + \vec{T}\Delta t = m\vec{v}'_A + m\vec{v}'_B$$

$$0 = m(v'_A)_t \cos 30° - m(v'_A)_n \sin 30° - mv'_B$$

$$0 = (0.5v_0)\cos 30° - (v'_A)_n \sin 30° - v'_B$$

$$0.5(v'_A)_n + v'_B = 0.433v_0$$

Sample Problem 13.16

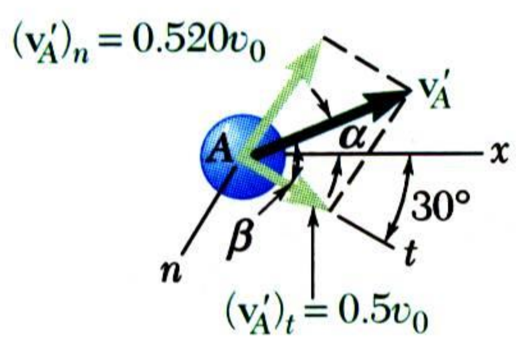


- The relative velocities along the line of action before and after the impact are related by the coefficient of restitution.

$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$
$$v'_B \sin 30° - (v'_A)_n = v_0 \cos 30° - 0$$
$$0.5v'_B - (v'_A)_n = 0.866v_0$$

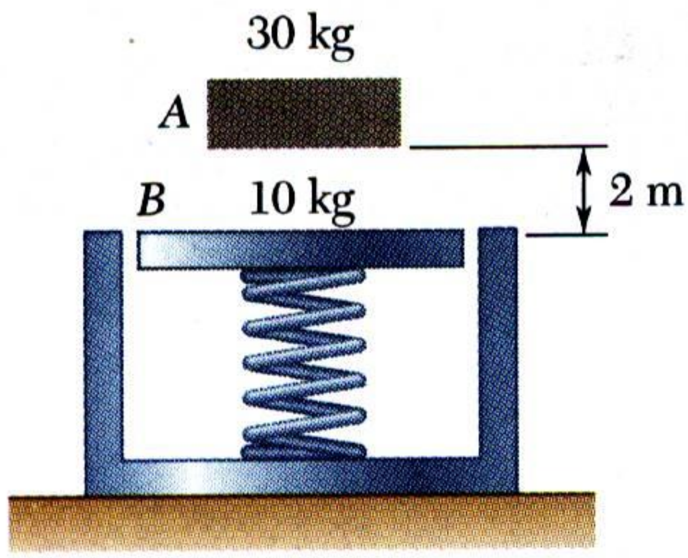
- Solve the last two expressions for the velocity of ball *A* along the line of action and the velocity of ball *B* which is horizontal.

$$(v'_A)_n = -0.520v_0 \qquad v'_B = 0.693v_0$$



$$\vec{v}'_A = 0.5v_0\vec{\lambda}_t - 0.520v_0\vec{\lambda}_n$$
$$v'_A = 0.721v_0 \quad \beta = \tan^{-1}\left(\frac{0.52}{0.5}\right) = 46.1°$$
$$\alpha = 46.1° - 30° = 16.1°$$
$$v'_B = 0.693v_0 \leftarrow$$

Sample Problem 13.17

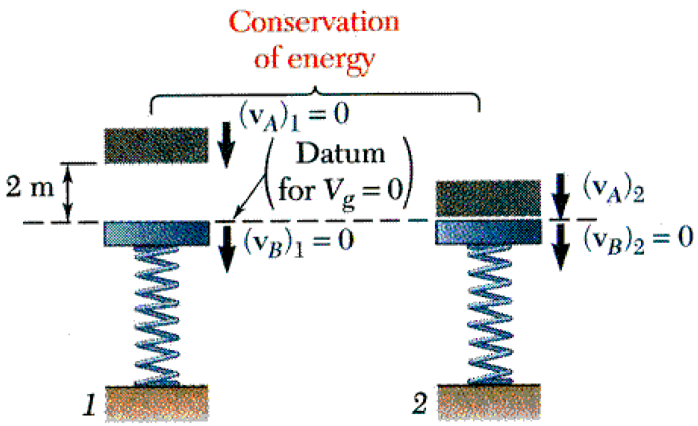


A 30 kg block is dropped from a height of 2 m onto the the 10 kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is k = 20 kN/m.

<u>SOLUTION</u>:

- Apply the principle of conservation of energy to determine the velocity of the block at the instant of impact.
- Since the impact is perfectly plastic, the block and pan move together at the same velocity after impact. Determine that velocity from the requirement that the total momentum of the block and pan is conserved.
- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

Sample Problem 13.17



SOLUTION:

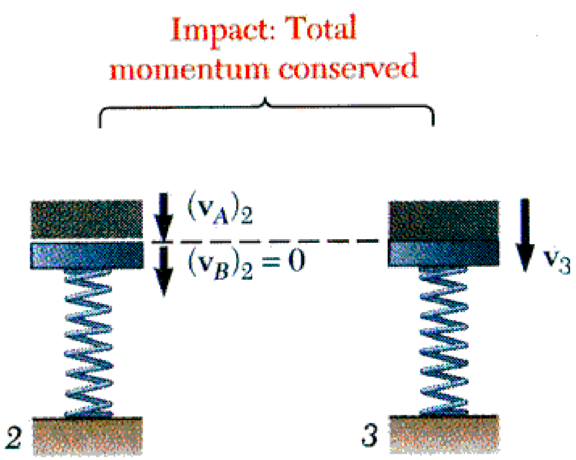
- Apply principle of conservation of energy to determine velocity of the block at instant of impact.

$$T_1 = 0 \qquad V_1 = W_A y = (30)(9.81)(2) = 588\text{ J}$$

$$T_2 = \tfrac{1}{2} m_A (v_A)_2^2 = \tfrac{1}{2}(30)(v_A)_2^2 \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 588\text{ J} = \tfrac{1}{2}(30)(v_A)_2^2 + 0 \qquad (v_A)_2 = 6.26\,\text{m/s}$$

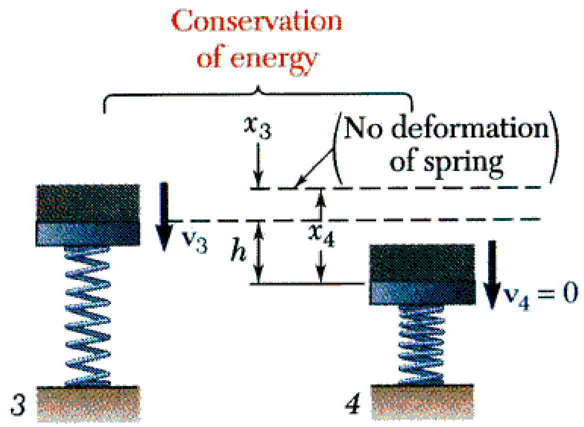


- Determine velocity after impact from requirement that total momentum of the block and pan is conserved.

$$m_A (v_A)_2 + m_B (v_B)_2 = (m_A + m_B) v_3$$

$$(30)(6.26) + 0 = (30 + 10) v_3 \qquad v_3 = 4.70\,\text{m/s}$$

Sample Problem 13.17



Initial spring deflection due to pan weight:

$$x_3 = \frac{W_B}{k} = \frac{(10)(9.81)}{20 \times 10^3} = 4.91 \times 10^{-3}\,\text{m}$$

- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

$$T_3 = \tfrac{1}{2}(m_A + m_B)v_3^2 = \tfrac{1}{2}(30 + 10)(4.7)^2 = 442\text{ J}$$

$$V_3 = V_g + V_e$$

$$= 0 + \tfrac{1}{2}kx_3^2 = \tfrac{1}{2}\left(20 \times 10^3\right)\left(4.91 \times 10^{-3}\right)^2 = 0.241\text{ J}$$

$$T_4 = 0$$

$$V_4 = V_g + V_e = (W_A + W_B)(-h) + \tfrac{1}{2}kx_4^2$$

$$= -392(x_4 - x_3) + \tfrac{1}{2}\left(20 \times 10^3\right)x_4^2$$

$$= -392\left(x_4 - 4.91 \times 10^{-3}\right) + \tfrac{1}{2}\left(20 \times 10^3\right)x_4^2$$

$$T_3 + V_3 = T_4 + V_4$$

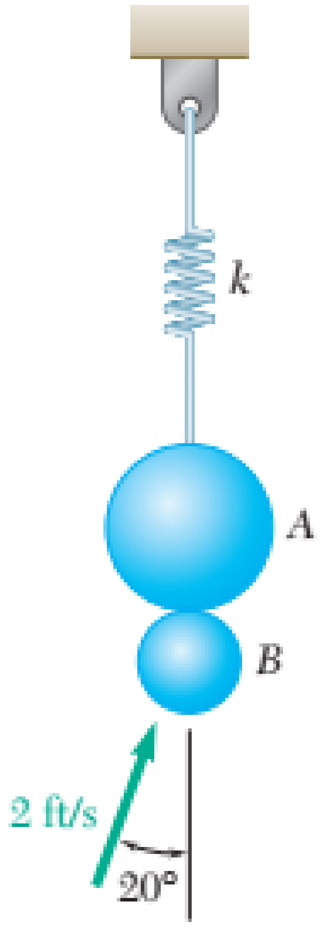
$$442 + 0.241 = 0 - 392\left(x_4 - 4.91 \times 10^{-3}\right) + \tfrac{1}{2}\left(20 \times 10^3\right)x_4^2$$

$$x_4 = 0.230\text{ m}$$

$$h = x_4 - x_3 = 0.230\text{ m} - 4.91 \times 10^{-3}\text{ m} \qquad \boxed{h = 0.225\text{ m}}$$

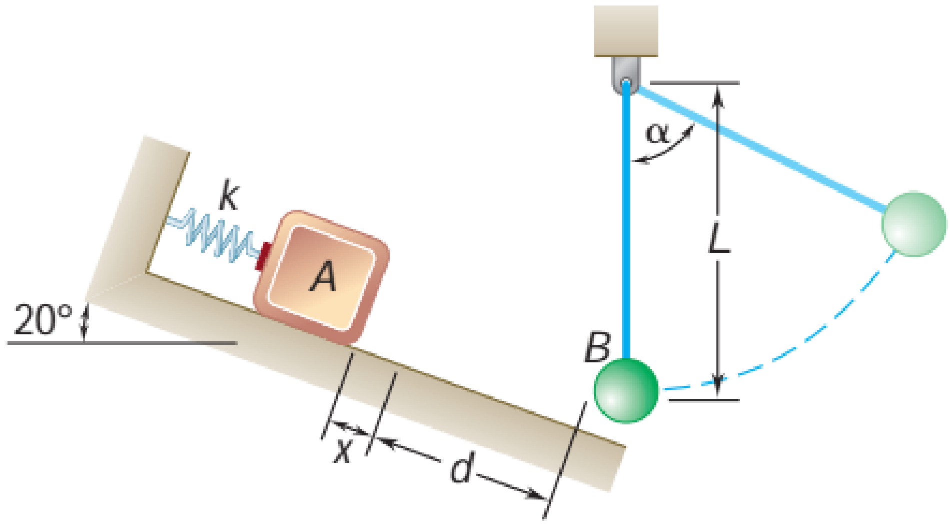
Prob # 13.184

A 2-lb ball *A* is suspended from a spring of constant 10 lb/in. and is initially at rest when it is struck by 1-lb ball *B* as shown. Neglecting friction and knowing the coefficient of restitution between the
balls is 0.6, determine
(*a*) the velocities of *A* and *B* after the impact,
(*b*) the maximum height reached by *A*.



Prob # 13.200

A 2-kg block *A* is pushed up against a spring compressing it a distance *x* 5 0.1 m. The block is then released from rest and slides down the 20° incline until it strikes a 1-kg sphere *B* which is suspended from a 1-m inextensible rope. The spring constant *k* =800 N/m, the coefficient of friction between *A* and the ground is 0.2, the distance *A* slides from the unstretched length of the spring, *d* =1.5 m, and the coefficient of restitution between *A* and *B* is 0.8. When α= 40°, determine
(*a*) the speed of *B,* (*b*) the tension in the rope.



Prob #13.188

When the rope is at an angle of a= 30° the 1-lb sphere *A* has a speed v_0 =4 ft/s. The coefficient of restitution between *A* and the 2-lb wedge *B* is 0.7 and the length of rope *l* = 2.6 ft. The spring constant has a value of 2 lb/in. and u = 20°. Determine
(*a*) the velocities of *A* and *B* immediately after the impact,
(*b*) the maximum deflection of the spring assuming *A* does not strike *B* again before this point.